Some aspects of a Chern-Simons-like coupling in an external magnetic field

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May 12, 2010

For a gauge theory which includes a light massive vector field interacting with the familiar photon $U(1)_{QED}$ via a Chern-Simons- like coupling, we study the static quantum potential. Our analysis is based on the gauge-invariant, but path-dependent, variables formalism. The result is that the theory describes an exactly screening phase. Interestingly enough, this result displays a marked departure of a qualitative nature from the axionic elctrodynamics result. However, the present result is analogous to that encountered in the coupling between the familiar photon $U(1)_{QED}$ and a second massive gauge field living in the so-called hidden-sector $U(1)_h$, inside a superconducting box.

PACS numbers: 11.15.-q; 11.10.Ef; 11.30.Cp
Keywords: gauge invariance, confinement, static potential

I. INTRODUCTION

Nowadays, one of the most actively pursued areas of research in physics consists of the investigation of extensions of the Standard Model (SM) such as axion-like particles and light extra hidden $U(1)$ gauge bosons, in order to explain cosmological and astrophysical results. These hidden $U(1)$ gauge bosons are frequently encountered in string theories physics. Also, this subject has had a revival after recent results of the PVLAS collaboration [1–13]. Nevertheless, although none of these searches ultimately has yielded a positive signal, the arguments in favor of the existence of axion-like particles or light extra hidden $U(1)$ gauge bosons, remain as cogent as ever.

In this perspective, it is useful to recall that the axion-like scenario can be qualitatively understood by the existence of light pseudoscalar bosons ϕ ("axions"), with a coupling to two photons. In other terms, the interaction term in the effective Lagrangian has the form $\mathcal{L}_I = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}\phi$, where $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$. However, the crucial feature of axionic electrodynamics is the mass generation due to the breaking of rotational invariance induced by a classical background configuration of the gauge field strength [14], which leads to confining potentials in the presence of nontrivial constant expectation values for the gauge field strength $F_{\mu\nu}$ [15]. In fact, in the case of a constant electric field strength expectation value the static potential remains Coulombic, while in the case of a constant magnetic field strength expectation value the potential energy is the sum of a Yukawa and a linear potential, leading to the confinement of static charges. Also it is important to point out that the magnetic character of the field strength expectation value needed to obtain confinement is in agreement with the current chromo-magnetic picture of the QCD vacuum [16]. In addition, similar results have been obtained in the context of the dual Ginzburg-Landau theory [17], as well as for a theory of antisymmetric tensor fields that results from the condensation of topological defects as a consequence of the Julia-Toulouse mechanism [18].

In a general perspective, we draw attention to the fact that much of this work has been inspired by studies coming from the realms of string theory [19, 20] and quantum field theory [21–28]. Indeed, as was observed in [29], the introduction of a second gauge field in addition to the usual photon was pioneered in Ref. [30], in the context of electrodynamics in the presence of magnetic monopoles [31]. Whereas the quantization for a system with two photons was later carried out in [32]. The possible existence of massive vector fields was also proposed in [33]. A kinetic term between the familiar photon $U(1)_{QED}$ and a second gauge field has also been considered in order to explain recent unexpected observations in high energy astrophysics [34].

We further note that recently another possible candidate for extensions of the SM has been studied [35]. It is the so-called Chern-Simons-like coupling scenario, which includes a light massive vector field interacting with the familiar photon $U(1)_{QED}$ via a Chern-Simons-like coupling. As a result, it was argued that this new model reproduces the effects of rotation of the polarization plane.

Given the relevance of these studies, it is of interest to

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improve our understanding of the physical consequences presented by this new scenario (Chern-Simons-like coupling scenario). Of special interest will be to study the connection or equivalence with the axion-like particles and light extra hidden $U(1)$ gauge bosons scenarios. Thus, our purpose here is to further explore the impact of a light massive vector field in the Chern-Simons-like coupling scenario on physical observables. To this end, we will study the screening and confinement issue. This issue is generally not discussed. Our calculation is accomplished by making use of the gauge-invariant but path-dependent variables formalism along the lines of Ref. [36–39], which is a physically-based alternative to the usual Wilson loop approach. As we shall see, in the case of a constant magnetic field the theory describes an exactly screening phase. This then implies that the static potential profile obtained from both the Chern-Simons-like coupling and axionic electrodynamics models are quite different. This means that the two theories are not equivalent. As it was shown in [15], axionic electrodynamics has a different structure which is reflected in a confining piece, which is not present in the Chern-Simons-like coupling scenario. Incidentally, the above static potential profile (Chern-Simons-like coupling scenario) is similar to that encountered in the coupling between the familiar massless electromagnetism $U(1)_{QED}$ and a hidden-sector $U(1)_h$ inside a superconducting box [40]. In this way one obtains a new connection among different models describing the same physical phenomena. In this connection, the benefit of considering the present approach is to provide unifications among different models, as well as exploiting the equivalence in explicit calculations, as we shall see in the course of the discussion.

II. INTERACTION ENERGY

We now discuss the interaction energy between static point-like sources for the model under consideration. To carry out such study, we will compute the expectation value of the energy operator H in the physical state $|\Phi\rangle$ describing the sources, which we will denote by $\langle H\rangle_\Phi$.

As anticipated above, the gauge theory we are considering describes the interaction between the familiar massive photon $U(1)_{QED}$ with a light massive vector field via a Chern-Simons- like coupling. In this case the corresponding theory is governed by the Lagrangian density [35]:

$$\mathcal{L} = -\frac{1}{4}F^2_{\mu\nu}(A) - \frac{1}{4}F^2_{\mu\nu}(B) + \frac{m^2_\gamma}{2}A^2_\mu + \frac{m^2_B}{2}B^2_\mu - \frac{\kappa}{2}\varepsilon^{\mu\nu\lambda\rho}A_\mu B_\nu F_{\lambda\rho}(A), \tag{2.1}$$

where m_γ is the mass of the photon, and m_B represents the mass for the gauge boson B. In particular, this alternative theory exhibits an effective mass for the component of the photon along the direction of the external magnetic field, exactly as it happens with axionic electrodynamics. If we consider the model in the limit of a very heavy B-field ($m_B \gg m_\gamma$) and we are bound to energies much below m_B, we are allowed to integrate over B_μ and to speak about an effective model for the A_μ-field. Then, the first crucial point is that by eliminating the gauge field B_μ in terms of A_μ in the original Lagrangian (2.1) one gets an effective Lagrangian $\mathcal{L}_{eff}$. This is accomplished by making use of the following shifting for the B_μ-field:

$$B_\mu \equiv \tilde{B}_\mu + \frac{\kappa}{2}\frac{1}{(\Delta+m^2_B)}\left[\eta_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{m^2_B}\right]\varepsilon^{\mu\lambda\rho\nu}A_\mu F_{\lambda\rho}(A)\,. \tag{2.2}$$

Once this is done, and dropping the constant factor which emerges by integrating out the $\tilde{B}_\mu$-field, we arrive at the following effective Lagrangian density

$$\begin{aligned}\mathcal{L}_{eff} = &-\frac{1}{4}F^2_{\mu\nu} + \frac{m^2_\gamma}{2}A_\mu A^\mu + \frac{\kappa^2}{4}A_\alpha F_{\beta\gamma}\frac{1}{(\Delta+m^2_B)}A^\alpha F^{\beta\gamma} \\ &+ \frac{\kappa^2}{2}A_\alpha F_{\beta\gamma}\frac{1}{(\Delta+m^2_B)}A^\gamma F^{\alpha\beta} \\ &+ \frac{\kappa^2}{8}\tilde{F}^{\alpha\beta}F_{\alpha\beta}\frac{1}{m^2_B(\Delta+m^2_B)}\tilde{F}^{\gamma\delta}F_{\gamma\delta},\end{aligned} \tag{2.3}$$

where $\widetilde{F}_{\mu\nu} \equiv {}^1\!/_2\varepsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$. The same result can be obtained by integrating out the B field in a path integral formulation of the model. The integral is gaussian in B and can be exactly computed leading to the effective Lagrangian (2.3).

Before going ahead, we would like to note that from Eq. (2.3) the gauge invariance is broken and one could argue about the possibility of getting a gauge invariant result for the static potential between test charges from (2.3). There are at least two available options to solve this apparent inconsistency. One way is to restore gauge invariance by inserting Stuckelberg compensating fields either into (2.1) or into (2.3). Once the compensators are integrated out the resulting model is explicitly gauge invariant. Unfortunately, this procedure introduces nonlocal effective interaction terms which are difficult to handle. In alternative we shall follow the Hamiltonian formulation discussed below.

As a second point, if we wish to study quantum properties of the electromagnetic field in the presence of external electric and magnetic fields, we should split the A_μ-field as the sum of a classical background, $\langle A_\mu\rangle$, and a small quantum fluctuation, a_μ,

$$A_\mu = \langle A_\mu\rangle + a_\mu. \tag{2.4}$$

Therefore the previous Lagrangian density, up to quadratic terms in the fluctuations, is also expressed as

$$
\begin{aligned}
\mathcal{L}_{eff} = & -\frac{1}{4} f_{\mu\nu}\Omega f^{\mu\nu} + \frac{1}{2} a_\mu M^2 a^\mu - \frac{\kappa^2}{2} f_{\gamma\beta}\left\langle A^\gamma\right\rangle \frac{1}{\left(\Delta + m_B^2\right)}\left\langle A_\alpha\right\rangle f^{\alpha\beta} + \frac{\kappa^2}{8} f_{\mu\nu} v^{\mu\nu}\frac{1}{m_B^2\left(\Delta + m_B^2\right)} v^{\lambda\rho} f_{\lambda\rho} \\
& - \kappa^2\left(\varepsilon^{jk0i} v_{0i}\left\langle A_j\right\rangle a^m \frac{1}{\left(\Delta + m_B^2\right)} f_{km}\right) + \kappa^2\left(\varepsilon^{jk0i} v_{0i}\left\langle A^m\right\rangle a_k \frac{1}{\left(\Delta + m_B^2\right)} f_{jm}\right) \\
& - \frac{\kappa^2}{2}\left(\varepsilon^{jk0i} v_{0i}\left\langle A^l\right\rangle a_l \frac{1}{\left(\Delta + m_B^2\right)} f_{jk}\right), \qquad (2.5)
\end{aligned}
$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, and $\Delta \equiv \partial_\mu \partial^\mu$. $\Omega \equiv 1 - \kappa^2 \frac{\left\langle A^i\right\rangle\left\langle A_i\right\rangle}{\left(\Delta + m_B^2\right)}$, and $M^2 \equiv m_\gamma^2 + \frac{\kappa^2}{2}\frac{v_{0i}v^{0i}}{\left(\Delta + m_B^2\right)}$. In the above Lagrangian we have considered the $v^{0i} \neq 0$ and $v^{ij} = 0$ case (referred to as the magnetic one in what follows), and simplified our notation by setting $\varepsilon^{\mu\nu\alpha\beta}\left\langle F_{\mu\nu}\right\rangle \equiv v^{\alpha\beta}$. As a consequence, the Lagrangian (2.5) becomes a Maxwell-Proca-like theory with a manifestly Lorentz violating term.

This effective theory provide us with a suitable starting point to study the interaction energy. However, before proceeding with the determination of the energy, it is necessary to restore the gauge invariance in (2.5). For this end we now carry out a Hamiltonian analysis. The canonically conjugate momenta are found to be $\Pi^\mu = -\left[1 - \kappa^2 \frac{\left\langle A^k\right\rangle\left\langle A_k\right\rangle}{\left(\Delta + m_B^2\right)}\right] f^{0\mu} + \kappa^2 \frac{\left\langle A^\mu\right\rangle\left\langle A_i\right\rangle}{\left(\Delta + m_B^2\right)} f^{i0} - \frac{\kappa^2}{4m_B^2} v^{\mu 0}\frac{1}{\left(\Delta + m_B^2\right)} v^{0i} f_{0i}$. This yields the usual primary constraint $\Pi^0 = 0$, while the momenta are $\Pi^i = -\left[1 - \kappa^2 \frac{\left\langle A^k\right\rangle\left\langle A_k\right\rangle}{\left(\Delta + m_B^2\right)}\right] f^{0i} + \kappa^2 \frac{\left\langle A^i\right\rangle\left\langle A_k\right\rangle}{\left(\Delta + m_B^2\right)} f^{k0} - \frac{\kappa^2}{4m_B^2} v^{i0}\frac{1}{\left(\Delta + m_B^2\right)} v^{0k} f_{0k}$. This leads us to the canonical Hamiltonian,

$$
\begin{aligned}
H_C = & -\int d^3x a_0\left(\partial_i \Pi^i + \frac{1}{2}\left\{m_\gamma^2 - \frac{\kappa^2 \mathbf{v}^2/2}{\left(\Delta + m_B^2\right)}\right\} a^0\right) + \int d^3x\left(-\frac{1}{2} a_i\left\{m_\gamma^2 - \frac{\kappa^2 \mathbf{v}^2/2}{\left(\Delta + m_B^2\right)}\right\} a^i\right) \\
& + \int d^3x\left(\frac{1}{2}\Pi_i\left\{1 - \frac{\kappa^2\left\langle \mathbf{A}\right\rangle^2}{\left(\Delta + m_B^2\right)}\right\}\Pi^i\right) + \int d^3x\left(\frac{\kappa^2}{2}\frac{1}{\left(\Delta + m_B^2\right)}\left(\left\langle \mathbf{A}\right\rangle \cdot \mathbf{\Pi}\right)^2\right) \\
& + \int d^3x\left(\frac{1}{2} B_i\left\{1 + \frac{\kappa^2\left\langle \mathbf{A}\right\rangle^2}{\left(\Delta + m_B^2\right)}\right\} B^i\right) + \int d^3x\left\{-\frac{1}{2}\frac{\kappa^2}{\left(\Delta + m_B^2\right)}\left(\varepsilon_{ikj}\left\langle A^k\right\rangle B^j\right)^2\right\} \\
& + \int d^3x\left(\kappa^2 \varepsilon^{jk0i} v_{0i}\left\langle A_j\right\rangle a^m \frac{1}{\left(\Delta + m_B^2\right)} f_{km}\right) - \int d^3x\left(\kappa^2 \varepsilon^{jk0i} v_{0i}\left\langle A^m\right\rangle a_k \frac{1}{\left(\Delta + m_B^2\right)} f_{jm}\right) \\
& + \int d^3x\left(\frac{\kappa^2}{2}\varepsilon^{jk0i} v_{0i}\left\langle A^l\right\rangle a_l \frac{1}{\left(\Delta + m_B^2\right)} f_{jk}\right), \qquad (2.6)
\end{aligned}
$$

where $\mathbf{v}$ stands for the external magnetic field (v^{0i}) and B^i is now the magnetic field associated to the fluctuation, namely, $B^i \equiv \epsilon^{ijk} f_{jk}$.

Time conservation of the primary constraint $\Pi_0 = 0$ yields the following secondary constraint $\Gamma(x) \equiv \partial_i \Pi^i + \left(m_\gamma^2 - \frac{\kappa^2 \mathbf{v}^2}{2}\frac{1}{\left(\Delta + m_B^2\right)}\right) a^0 = 0$. Notice that the nonvanishing bracket $\left\{\Pi^0, \partial_i \Pi^i + \left(m_\gamma^2 - \frac{\kappa^2 \mathbf{v}^2}{2}\frac{1}{\left(\Delta + m_B^2\right)}\right) a^0\right\}$ shows that the above pair of constraints are second class constraints, as expected for a theory with an explicit mass term which breaks the gauge invariance. To convert the second class system into first class we enlarge the original phase space by introducing a canonical pair of fields θ and Π_θ [38]. It follows, therefore, that a new set of first class constraints can be defined in this extended space:

$$
\Lambda_1 = \Pi_0 + \left(m_\gamma^2 - \frac{\kappa^2 \mathbf{v}^2}{2}\frac{1}{\left(\Delta + m_B^2\right)}\right) a^0, \qquad (2.7)
$$

and

$$
\Lambda_2 \equiv \Gamma + \Pi_\theta. \qquad (2.8)
$$

In this way the gauge symmetry of the theory under consideration has been restored. Then, the new effective Lagrangian, after integrating out the θ-field, becomes

$$\mathcal{L}_{eff} = -\frac{1}{4} f_{\mu\nu} \left[\frac{\left(\Delta^2 + a^2\Delta + b^2\right)}{\Delta\left(\Delta + m_B^2\right)} \right] f^{\mu\nu} - \left\langle A^i \right\rangle f_{i0} \frac{1}{\left(\Delta + m_B^2\right)} \left\langle A_k \right\rangle f^{k0} - \frac{\kappa^2}{2} f_{ki} \left\langle A^k \right\rangle \frac{1}{\left(\Delta + m_B^2\right)} \left\langle A_l \right\rangle f^{li} + \frac{\kappa^2}{8} v^{0i} f_{0i} \frac{1}{m_B^2\left(\Delta + m_B^2\right)} v^{0k} f_{0k}, \tag{2.9}$$

where $a^2 \equiv m_B^2 + m_\gamma^2 \left(1 - \kappa^2 \frac{\langle A_k\rangle\langle A^k\rangle}{m_\gamma^2}\right)$, and $b^2 = m_\gamma^2 \left(m_B^2 - \frac{\kappa^2 \mathbf{v}^2}{2m_\gamma^2}\right)$. We observe that to get the above theory we have ignored the last three terms in (2.6) because it add nothing to the static potential calculation, as we will show it below. In other words, the new effective action (2.6) provide us with a suitable starting point to study the interaction energy without loss of physical content.

We now turn our attention to the calculation of the interaction energy. In order to obtain the corresponding Hamiltonian, the canonical quantization of this theory from the Hamiltonian analysis point of view is straightforward and follows closely that of our previous work [38, 39]. The canonical momenta read $\Pi^\mu = -\left(\frac{\Delta^2+a^2\Delta+b^2}{\Delta\left(\Delta+m_B^2\right)}\right) f^{0\mu} + \kappa^2 \frac{\langle A^\mu\rangle\langle A_k\rangle}{\left(\Delta+m_B^2\right)} f^{k0} + \frac{\kappa^2}{4}\frac{v^{0\mu}}{m_B^2}\frac{1}{\left(\Delta+m_B^2\right)} v^{0k} f_{0k}$, and one immediately identifies the usual primary constraint $\Pi^i = -\left(\frac{\Delta^2+a^2\Delta+b^2}{\Delta\left(\Delta+m_B^2\right)}\right) f^{0i} + \kappa^2 \frac{\langle A^i\rangle\langle A_k\rangle}{\left(\Delta+m_B^2\right)} f^{k0} + \frac{\kappa^2}{4}\frac{v^{0i}}{m_B^2}\frac{1}{\left(\Delta+m_B^2\right)} v^{0k} f_{0k}$. The canonical Hamiltonian is thus given by

$$H_C = \int d^3x \left\{ -a_0 \partial_i \Pi^i + \frac{1}{2} B^i \frac{\left(\Delta^2 + a^2\Delta + b^2\right)}{\Delta\left(\Delta + m_B^2\right)} B^i \right\} - \frac{1}{2} \int d^3x \Pi_i \frac{\left(\Delta + m_B^2\right)}{\left(\Delta + a^2 + \frac{b^2}{\Delta}\right)} \Pi^i + \frac{\kappa^2}{2} \int d^3x \Pi_i \left\langle A^i \right\rangle \frac{\left(\Delta + m_B^2\right)}{\left(\Delta + a^2 + \frac{b^2}{\Delta}\right)^2} \left\langle A^k \right\rangle \Pi_k + \frac{\kappa^2}{2} \int d^3x f_{ki} \left\langle A^k \right\rangle \frac{1}{\left(\Delta + m_B^2\right)} \left\langle A_l \right\rangle f^{li} + \frac{\kappa^2 \mathbf{v}^2}{8 m_B^2} \int d^3x \Pi_i \frac{\left(\Delta + m_B^2\right)}{\left(\Delta + a^2 + \frac{b^2}{\Delta}\right)^2} \Pi^i, \tag{2.10}$$

where $a^2 = m_B^2 + m_\gamma^2 + \kappa^2 \left\langle \mathbf{A} \right\rangle^2$ and $b^2 = m_\gamma^2 m_B^2 + \frac{\kappa^2}{2}\mathbf{v}^2$. Since our energies are all much below m_B, it is consistent with our considerations to neglect $\kappa^2 \left\langle \mathbf{A} \right\rangle^2$ with respect to m_B^2. This implies that a^2 and b^2 should be taken as: $a^2 = m_B^2$ and $b^2 = m_\gamma^2 m_B^2 + \frac{\kappa^2}{2}\mathbf{v}^2$.

The consistency condition $\dot{\Pi}_0 = 0$ leads to the usual Gauss constraint $\Gamma_1(x) \equiv \partial_i \Pi^i = 0$. It is also possible to verify that no further constraints are generated by this theory. Consequently, the extended Hamiltonian that generates translations in time then reads $H = H_C + \int d^2x \left(c_0(x)\Pi_0(x) + c_1(x)\Gamma_1(x)\right)$, where $c_0(x)$ and $c_1(x)$ are arbitrary Lagrange multipliers. Moreover, it follows from this Hamiltonian that $\dot{a}_0(x) = \left[a_0(x), H\right] = c_0(x)$, which is completely arbitrary. Since $\Pi^0 = 0$ always, neither a^0 and Π^0 are of interest in describing the system and may be discarded from the theory. If a new arbitrary coefficient $c(x) = c_1(x) - A_0(x)$ is introduced the Hamiltonian may be rewritten as

$$H = \int d^3x \left\{ c(x) \partial_i \Pi^i + \frac{1}{2} B^i \frac{\left(\Delta^2 + a^2\Delta + b^2\right)}{\Delta\left(\Delta + m_B^2\right)} B^i \right\} - \frac{1}{2} \int d^3x \Pi_i \frac{\left(\Delta + m_B^2\right)}{\left(\Delta + a^2 + \frac{b^2}{\Delta}\right)} \Pi^i + \frac{\kappa^2}{2} \int d^3x \Pi_i \left\langle A^i \right\rangle \frac{\left(\Delta + m_B^2\right)}{\left(\Delta + a^2 + \frac{b^2}{\Delta}\right)^2} \left\langle A^k \right\rangle \Pi_k + \frac{\kappa^2}{2} \int d^3x f_{ki} \left\langle A^k \right\rangle \frac{1}{\left(\Delta + m_B^2\right)} \left\langle A_l \right\rangle f^{li} + \frac{\kappa^2 \mathbf{v}^2}{8 m_B^2} \int d^3x \Pi_i \frac{\left(\Delta + m_B^2\right)}{\left(\Delta + a^2 + \frac{b^2}{\Delta}\right)^2} \Pi^i. \tag{2.11}$$

We can at this stage impose a subsidiary on the vector potential such that the full set of constraints become

second class. A particularly convenient choice is found to be

$$\Gamma_2(x) \equiv \int_{C_{\xi x}} dz^\nu a_\nu(z) \equiv \int_0^1 d\lambda x^i a_i(\lambda x) = 0, \quad (2.12)$$

where λ ($0 \leq \lambda \leq 1$) is the parameter describing the spacelike straight path $x^i = \xi^i + \lambda(x-\xi)^i$, and ξ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. The choice (2.12) leads to the Poincaré gauge [41]. As a consequence, we can now write down the only nonvanishing Dirac bracket for the canonical variables

$$\left\{a_i(x), \Pi^j(y)\right\}^* = \delta_i^j \delta^{(2)}(x-y) - \partial_i^x \int_0^1 d\lambda x^j \delta^{(2)}(\lambda x - y). \quad (2.13)$$

At this point, we have all the elements necessary to find the interaction energy between point-like sources for the model under consideration. As we have already indicated, we will calculate the expectation value of the energy operator H in the physical state $|\Phi\rangle$. In this context, we recall that the physical state $|\Phi\rangle$ can be written as

$$|\Phi\rangle \equiv \left|\overline{\Psi}(\mathbf{y})\Psi(\mathbf{0})\right\rangle = \overline{\psi}(\mathbf{y}) \exp\left(iq\int_{\mathbf{0}}^{\mathbf{y}} dz^i a_i(z)\right)\psi(\mathbf{0})|0\rangle, \quad (2.14)$$

where $|0\rangle$ is the physical vacuum state. The line integral is along a spacelike path starting at $\mathbf{0}$ and ending at $\mathbf{y}$, on a fixed time slice. Notice that the charged matter field together with the electromagnetic cloud (dressing) which surrounds it, is given by $\Psi(\mathbf{y}) = \exp\left(-iq\int_{C_{\xi \mathbf{y}}} dz^\mu A_\mu(z)\right)\psi(\mathbf{y})$. Thanks to our path choice, this physical fermion then becomes $\Psi(\mathbf{y}) = \exp\left(-iq\int_{\mathbf{0}}^{\mathbf{y}} dz^i A_i(z)\right) \times \psi(\mathbf{y})$. In other terms, each of the states ($|\Phi\rangle$) represents a fermion-antifermion pair surrounded by a cloud of gauge fields to maintain gauge invariance.

Further, by taking into account the structure of the Hamiltonian above, we observe that

$$\Pi_i(x)\left|\overline{\Psi}(\mathbf{y})\Psi(\mathbf{y}')\right\rangle = \overline{\Psi}(\mathbf{y})\Psi(\mathbf{y}')\Pi_i(x)|0\rangle + q\int_{\mathbf{y}}^{\mathbf{y}'} dz_i \delta^{(3)}(\mathbf{z}-\mathbf{x})|\Phi\rangle. \quad (2.15)$$

Having made this observation and since the fermions are taken to be infinitely massive (static sources), we can substitute Δ by $-\nabla^2$ in Eq. (2.11). Therefore, the expectation value $\langle H\rangle_\Phi$ is expressed as

$$\langle H\rangle_\Phi = \langle H\rangle_0 + \langle H\rangle_\Phi^{(1)} + \langle H\rangle_\Phi^{(2)}, \quad (2.16)$$

where $\langle H\rangle_0 = \langle 0|H|0\rangle$. The $\langle H\rangle_\Phi^{(1)}$ and $\langle H\rangle_\Phi^{(2)}$ terms are given by

$$\langle H\rangle_\Phi^{(1)} = -\frac{b^2 B}{2}\int d^3x \langle\Phi|\Pi_i \frac{\nabla^2}{(\nabla^2 - M_1^2)}\Pi^i|\Phi\rangle + \frac{b^2 B}{2}\int d^3x \langle\Phi|\Pi_i \frac{M_2^2}{M_1^2}\frac{\nabla^2}{(\nabla^2 - M_2^2)}\Pi^i|\Phi\rangle, \quad (2.17)$$

and

$$\langle H\rangle_\Phi^{(2)} = m_B^2 b^2 B\int d^3x \langle\Phi|\Pi_i \frac{1}{(\nabla^2 - M_1^2)}\Pi^i|\Phi\rangle - m_B^2 b^2 B\int d^3x \langle\Phi|\Pi_i \frac{M_2^2}{M_1^2}\frac{1}{(\nabla^2 - M_2^2)}\Pi^i|\Phi\rangle, \quad (2.18)$$

where $B = \frac{1}{M_2^2(M_2^2 - M_1^2)}$, $M_1^2 \equiv \frac{a^2}{2}\left[1+\sqrt{1-\frac{4b^2}{a^4}}\right]$, and $M_2^2 \equiv \frac{a^2}{2}\left[1-\sqrt{1-\frac{4b^2}{a^4}}\right]$.

We have neglected the terms in (2.11) where $\left(\Delta + a^2 + \frac{b^2}{\Delta}\right)^2$ appears in the denominator, the reason being that we wish to compute an interparticle potential, which expresses the effects of photons exchange in the low-energy (or low-frequency) limit. Therefore, these terms we are mentioning are suppressed in view of the presence of higher power of the frequency in the denominator. Another important point to be highlighted in our discussion comes from the expressions for M_1^2 and M_2^2. Our treatment is only valid under the assumption that $a^4 > 4b^2$. However, this condition is equivalent to taking $\kappa^2 \mathbf{v}^2 < \frac{m_B^4}{2}$, which is perfectly compatible with our approximation. So, we restrict ourselves to an external magnetic field such that $|\mathbf{v}| < \frac{m_B^2}{2\kappa^2}$.

Using Eq. (2.15), the $\langle H\rangle_\Phi^{(1)}$ and $\langle H\rangle_\Phi^{(2)}$ terms can be rewritten as

$$\langle H\rangle_{\Phi}^{(1)} = -\frac{b^2 Bq^2}{2}\int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i' \delta^{(3)}\left(\mathbf{x}-\mathbf{z}'\right)\frac{\nabla^2}{\left(\nabla^2 - M_1^2\right)} \times \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \delta^{(3)}\left(\mathbf{x}-\mathbf{z}\right)$$
$$+ \frac{b^2 Bq^2}{2}\frac{M_2^2}{M_1^2}\int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i' \delta^{(3)}\left(\mathbf{x}-\mathbf{z}'\right)\frac{\nabla^2}{\left(\nabla^2 - M_2^2\right)} \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \delta^{(3)}\left(\mathbf{x}-\mathbf{z}\right), \tag{2.19}$$

and

$$\langle H\rangle_{\Phi}^{(2)} = m_B^2 b^2 Bq^2 \int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i' \delta^{3}\left(\mathbf{x}-\mathbf{z}'\right)\frac{1}{\left(\nabla^2 - M_1^2\right)} \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \delta^{(3)}\left(\mathbf{x}-\mathbf{z}\right)$$
$$- \frac{m_B^2 b^2 Bq^2 M_2^2}{M_1^2}\int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i' \delta^{(3)}\left(\mathbf{x}-\mathbf{z}'\right)\frac{1}{\left(\nabla^2 - M_2^2\right)} \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \delta^{(3)}\left(\mathbf{x}-\mathbf{z}\right). \tag{2.20}$$

Following our earlier procedure [36, 39], we see that the potential for two opposite charges located at $\mathbf{y}$ and $\mathbf{y}'$ takes the form

$$V = -\frac{q^2}{4\pi}\frac{1}{a^2\sqrt{1-4b^2/a^4}} \times \left[\left(M_1^2 - m_B^2\right)\frac{e^{-M_1 L}}{L} - \left(M_2^2 - m_B^2\right)\frac{e^{-M_2 L}}{L}\right]. \tag{2.21}$$

Consequently, our analysis reveals that the theory under consideration describes an exactly screening phase. It is important to realize that expression (2.21) displays a marked departure of a qualitative nature from the result from axionic elctrodynamics. As already mentioned, axionic electrodynamics has a different structure which is reflected in a confining piece, which is not present in the Chern-Simons-like coupling scenario. It is to be noted that the choice of the gauge is in this development really arbitrary. Put another way, being the formalism completely gauge invariant, we would obtain exactly the same result in any gauge. We also note here that by considering the limit $b \to 0$, we obtain a theory of two independent uncoupled $U(1)$ gauge bosons, one of which is massless. In such a case, one can easily verify that the static potential is a Yukawa-like correction to the usual static Coulomb potential.

Finally, the following remark is pertinent at this point. It should be noted that by substituting B_μ by $\partial_\mu\phi$ in (2.1), the theory under consideration assumes the form [42]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{m_\gamma^2}{2} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\kappa}{2m_B}\varepsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\phi, \tag{2.22}$$

which is similar to axionic electrodynamics. In fact, it is worth recalling here that axionic electrodynamics is described by [15]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g}{8}\phi\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m_A^2}{2}\phi^2, \tag{2.23}$$

hence we see that both theories are quite different.

Thus, after performing the integration over ϕ in (2.22), the effective Lagrangian density reads

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{m_\gamma^2}{2}A_\mu^2 - \frac{\kappa^2}{8m_B^2}\varepsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\frac{1}{\nabla^2} \times \varepsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}. \tag{2.24}$$

This expression can now be rewritten as

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}^2 + \frac{m_\gamma^2}{2}a_\mu^2 - \frac{\kappa^2}{2m_B^2}\varepsilon^{\mu\nu\alpha\beta}\left\langle F_{\mu\nu}\right\rangle\varepsilon^{\lambda\rho\gamma\delta}\left\langle F_{\lambda\rho}\right\rangle \times f_{\alpha\beta}\frac{1}{\nabla^2}f_{\gamma\delta}, \tag{2.25}$$

where $\langle F_{\mu\nu}\rangle$ represents the constant classical background. Here $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ describes fluctuations around the background. The above Lagrangian arose after using $\varepsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}\rangle\langle F_{\alpha\beta}\rangle = 0$ (which holds for a pure electric or a pure magnetic background). By introducing the notation $\varepsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}\rangle \equiv v^{\alpha\beta}$ and $\varepsilon^{\rho\sigma\gamma\delta}\langle F_{\rho\sigma}\rangle \equiv v^{\gamma\delta}$, expression (2.25) then becomes

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}^2 + \frac{m_\gamma^2}{2}a_\mu^2 - \frac{\kappa^2}{2m_B^2}v^{\alpha\beta}f_{\alpha\beta}\frac{1}{\nabla^2}v^{\gamma\delta}f_{\gamma\delta}, \tag{2.26}$$

where the tensor $v^{\alpha\beta}$ is not arbitrary, but satisfies $\varepsilon^{\mu\nu\alpha\beta}v_{\mu\nu}v_{\alpha\beta} = 0$.

Following the same steps employed for obtaining (2.21), the static potential is expressed as

$$V = -\frac{q^2}{4\pi}\frac{e^{-ML}}{L}, \tag{2.27}$$

with $M^2 \equiv m_\gamma^2 + \frac{\kappa^2}{m_B^2}\mathbf{v}^2$. Again, the theory describes a screening phase, as we have just seen above.

III. FINAL REMARKS

Let us summarize our work. Once again we have advocared a key point for understanding the physical

content of gauge theories, that is, the identification of field degrees of freedom with observable quantities. We have showed that the static potential profile obtained from both a gauge theory which includes a light massive vector field interacting with the familiar photon $U(1)_{QED}$ via a Chern-Simons-like coupling and axionic electrodynamics models are quite different. This means that the two theories are not equivalent. As it was shown in [15], axionic electrodynamics has a different structure which is reflected in a confining piece, which is not present in the gauge theory which includes a light massive vector field interacting with the familiar photon $U(1)_{QED}$ via a Chern-Simons-like coupling. However, our result is analogous to that encountered in the coupling between the familiar photon $U(1)_{QED}$ and a second massive gauge field living in the so-called hidden-sector $U(1)_h$, inside a superconducting box.

IV. ACKNOWLEDGMENTS

One of us (PG) wants to thank the Field Theory Group of the CBPF and the Physics Department of the Università di Trieste for hospitality. This work was supported in part by Fondecyt (Chile) grant 1080260 (PG). (J.H-N) expresses his gratitude to CNPq.

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